tion; the small perturbation parameter was $\epsilon = \sin\alpha$, where α is the half apical angle. The solution to the zero equation is that for a circular, cylindrical shell which was presented in Ref. 1. Solutions for the correction terms $\epsilon\sigma_1$ and $\epsilon^2\sigma_2$ are not uniformly valid for large τ because these solutions increase with increasing τ. However, these formulas are a good approximation when the quantity $\epsilon\tau$ is sufficiently small; e.g., see Ref. 3.

The bending stress at the radially outward shell surface at the clamped support for $\alpha = 10°$ and $a/h = 10$ is presented in Fig. 2. The conical response data indicate that one correction term is adequate to describe the conical shell response when $\tau < 4$, during which one relative maximum and one relative minimum are reached. It is only when $\tau > 4$ that the second correction term makes any noticeable difference in the response. These data also indicate that the bending stress history for the conical shell closely follows the bending stress history for the circular, cylindrical shell.

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Determination of Plasma Drift Velocity by an Ion Acoustic Wave Technique

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THIS article outlines a method that uses the transmission of ion acoustic waves to determine the drift velocity of a streaming, collisionless N_2^+-plasma, which approximates the ionospheric conditions of ion mass and plasma density.

In a drifting, collisionless plasma, the total ion acoustic wave velocity is the sum of the wave propagation velocity and the plasma drift velocity; i.e., $\nu_w = \nu_p + \nu_d$. Therefore, if ν_p is known, the plasma drift velocity can be obtained directly from acoustic wave time-of-flight measurements.

The propagation velocity of longitudinal, ion acoustic waves, as described by Spitzer,[1] is related to other plasma parameters by the equation

$$\nu_p^2 = (Z\gamma_e kT_e + \gamma_i kT_i)/m_i \quad (1)$$

where Z is the ionic charge, γ_e and γ_i are specific heat ratios, k is Boltzmann's constant, T_e and T_i are electron and ion temperatures, and m_i is the ion mass. Since ion acoustic waves undergo severe Landau damping unless $T_e \gg T_i$, Eq. (1) simplifies to

$$\nu_p^2 = Z\gamma_e kT_e/m_i \quad (2)$$

which applies to most physical cases of interest.

The method of finding ν_d will depend on the ratio, $\nu_d:\nu_p$, which, from Eq. (2), is seen to be partially determined by the electron temperature.[2] For the condition that $\nu_p \approx \nu_d$, the propagation velocity can be compensated for by measuring lapse times of waves traveling over known distances, parallel and perpendicular to the plasma stream.‡ In the event that $\nu_p \ll \nu_d$, only a direct measurement of ν_w would be required.

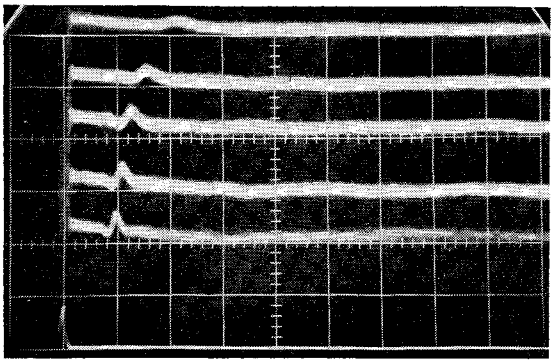

Fig. 1 Upper traces: receiver signals for drift velocities of 11.4 km/sec, 15.2 km/sec, 18.2 km/sec, 20.8 km/sec, and 23.2 km/sec. Transmitting distance, 38 cm: oscilloscope sweep, 17 μsec/cm. Lower trace: driving pulse to emitter.

To verify this concept, a streaming, collisionless, N_2^+ plasma was generated in a 4 × 8 ft, ultra-high vacuum chamber by a modified, 15 cm Kaufman ion thruster. Neutralization of the N_2^+ ions, produced and accelerated down the axis of the chamber by the thruster, was accomplished by an emissive tantalum wire. The Kaufman thruster was found to be ideal for this purpose in that it afforded a wide range of control over the plasma stream parameters. The plasma density, determined by a shielded Faraday cup, was varied between 10^5 and 10^6 ions/cm^3. The plasma drift velocity, which was calibrated with a 127° electrostatic energy analyzer,[3] was varied from 11.1 km/sec to 23.0 km/sec.

The ion acoustic waves were generated by driving a vertical, 0.03-in. stainless steel wire emitter with a 2-μsec, 50-v pulse at a repetition rate of 60 Hz. The waves were detected downstream by a receiving probe which consisted of a 1.5-in. square stainless steel mesh grid. The use of the wire emitter probe upstream was intended to minimize plasma wake effects in the experiment. The distance between the emitter and receiver probes, which were unbiased, was varied from 11 cm to 47 cm.

Examples of the data obtained are shown in Figs. 1 and 2. The bottom trace in both figures represents the driving pulse to the emitter. The upper traces of Fig. 1 display acoustic waves obtained at a fixed transmitting distance of 38 cm and five different drift velocities. The upper curves of Fig. 2 represent the opposite case where the drift velocity was held constant at 18.2 km/sec and acoustic waves obtained at three different transmitting distances.

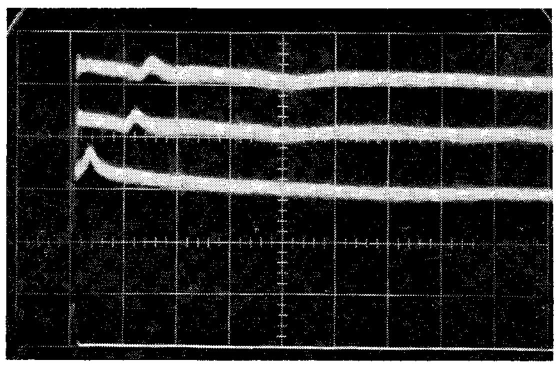

Fig. 2 Upper traces: receiver signals for transmitting distances of 47 cm, 38 cm, and 10.9 cm. Drift velocity: 18.2 km/sec; oscilloscope sweep, 17 μsec/cm. Lower trace: driving signal to emitter.

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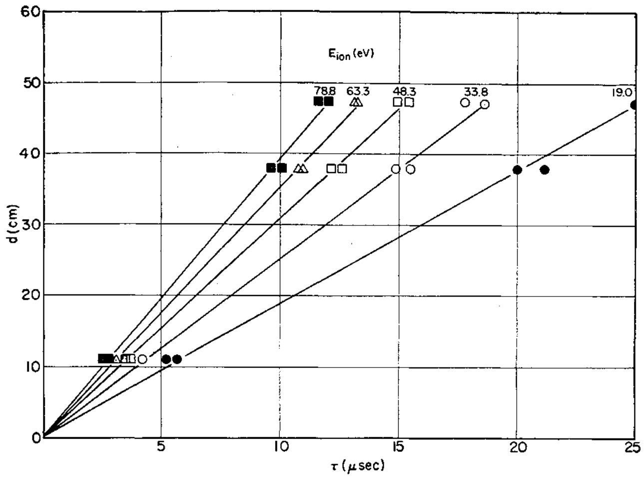

Fig. 3 Transmitting distance as function of laspse time for different plasma drift velocities.

In Fig. 3, the transmitting distances are plotted as functions of the lapse times for different velocities. The strict proportionality of the distance vs time proves that we have no noticeable influence of plasma sheath effects on the acoustic wave velocity ν_w given by the slope. Further, it was found that the mentioned variation of the plasma density did not affect the lapse time.

The acoustic wave velocities ν_w determined from the time-of-flight data shown in Fig. 3, are compared, in Fig. 4, with the ion drift velocities ν_d obtained from the electrostatic energy analyzer. The agreement of ν_w with ν_d indicates that the propagation velocity of the acoustic waves has a negligible effect in this range of velocities; i.e., $\nu_p \ll \nu_d$.

The described ion acoustic wave technique is particularly desirable for plasma drift velocity measurements in that it has been proved, within the limits of this experiment, to be independent of plasma density, does not depend on ion mass or charge, but rather, requires only the knowledge of the probe separation and wave lapse time. In addition to drift velocity measurements in the plasma wind tunnel, it is the intent of the authors to apply the technique, in a further stage of development, to the determination of orbital speeds of space vehicles in the ionosphere. Further investigations will be directed towards extension of the plasma parameter limits and the influence of the electron temperature on the propagation process.

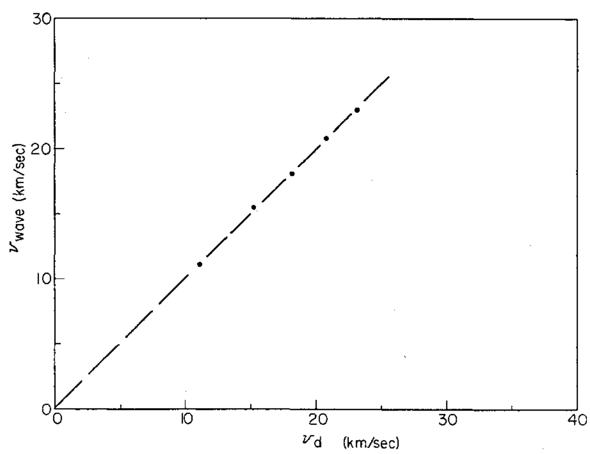

Fig. 4 Comparison of ion drift velocity ν_d and wave transmission velocity ν_w.

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Nonlinear Beam Vibration with Variable Axial Boundary Restraint

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Nomenclature

A = cross-sectional area of beam
D_Q = transverse shearing stiffness of beam, $\gamma = Q/D_Q$
E = Young's modulus
g = time-dependent lateral displacement amplitude function
h = time-dependent axial displacement amplitude function
i = time-dependent shear angle amplitude function
I = cross-sectional moment of inertia of beam
k_D = spring constant representing axial restraint
K = nondimensional spring constant parameter, $K = k_D l/AE$
K_1 = complete elliptic integral of the first kind
l = length of beam
Q = shearing force
r = radius of gyration of the cross section, $r = (I/A)^{1/2}$
t = time
u = axial displacement of beam midsurface
w = lateral displacement of beam midsurface
w_{MAX} = maximum lateral displacement
x,z = axial and lateral coordinates
α = nondimensional function of displacement, $\alpha = (g_{MAX}/r)$: (for example with hinged ends, $g_{MAX} = w_{MAX}$; for example with clamped ends, $g_{MAX} = w_{MAX}/2$)
γ = shearing strain in beam
ω_n = actual frequency of nth mode lateral vibration
ω_0 = frequency as given by elementary theory for the nth mode

An investigation[1] was undertaken to establish simultaneously the effects of transverse shear, rotatory inertia, and variable midplane stretching on the lateral vibration behavior of solid and sandwich beams. This investigation was the forerunner of a more extensive program aimed at establishing frequency prediction capability for internal structural members of aerospace vehicles. These structural members reflect the use of nonclassical boundary restraints.

After the study conducted in Ref. 1, other investigations[2,3] appeared that dealt, in different theoretical fashions, with the limiting case of infinite axial restraint but yielded identical or essentially identical results. It is felt that the method of analysis employed in Ref. 1 is more easily adaptable to real structures and provides a more direct approach to the prob-

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